Worksheet 8

1. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x$

    a. Is f injective (one-to-one)? If so, prove the statement (**Ch. 5.5**)

    b. Is f surjective (onto)? If so, prove the statement (**Ch. 5.5**)

2. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$

    a. Is f injective (one-to-one)? If so, prove the statement (**Ch. 5.5**)

    b. Is f surjective (onto)? If so, prove the statement (**Ch. 5.5**)

3. Let $f: \mathbb{N} \to \mathcal{P}(\mathbb{N})$ be defined by $f(n) = \{1, 2, 3, \ldots, n\}$

   a. Is f injective (one-to-one)? If so, prove the statement (**Ch. 5.5**)

   b. Is f surjective (onto)? If so, prove the statement (**Ch. 5.5**)

4. Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by $f\big((x, y)\big) = x + 2y$

   a. Is f injective (one-to-one)? If so, prove the statement (**Ch. 5.5**)

   b. Is f surjective (onto)? If so, prove the statement (**Ch. 5.5**)

5. Let $f: \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = n + 1$ (**Ch. 5.5**)

   a. Is f injective (one-to-one)? If so, prove the statement

   b. Is f surjective (onto)? If so, prove the statement

6. Consider $A = \{1,3,5\}$, $B = \{3,6,8\}$

   a. Define a function that takes elements in A to B

   b. Is the function you defined injective?

   c. Is the function you defined surjective?

7. Consider $A = \{1,3,5,7\}$, $B = \{3,6\}$

   a. Define a function that takes elements in A to B

   b. Is the function you defined injective?

   c. Is the function you defined surjective?

8. Consider $A = \{1,3\}$, $B = \{3,6,8,10\}$

    a. Define a function that takes elements in A to B

    b. Is the function you defined injective?

    c. Is the function you defined surjective?

From 6-8, what conjecture can you make about the injectivity and surjectivity regarding the size of domain and codomain?